Analyzing International Trade through Graph Theory

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Abstract

This work presents a simplified mathematical model of global trade as a directed network and uses random walks and stationary distributions to study long-run network centrality. Each row of the trade matrix is normalized into destination shares, so the resulting stationary distribution measures visitation centrality within the selected network rather than absolute trade volume or economic size. Using a recursive tariff-adjustment rule inspired by U.S. trade policy, the model examines how changes to U.S.-bound trade links alter these destination shares and the resulting centrality values over repeated cycles. The model is intended as a sensitivity analysis under explicit simplifying assumptions, not as an empirical forecast of future trade flows.

1 Introduction

International trade is the foundation of the global economy. Everyday, billions of dollars in goods are imported and exported from one country to another. From the raw materials in Africa to the manufactured goods in East Asia, this intricate network of trade is the backbone to economic growth and even shapes the foreign relationships between nations. In this ever so globalized world, the welfare of a nation often depend on international trade because no nation is economically isolated.

To analyze this complex network, researchers often turn to mathematics. Mathematical tools offer a structured approach to analyze massive amounts of complex data, helping researchers identify key players in the global commerce. When important events happen, mathematical analysis can even help predict the new characteristics within this complex network.

Since global trade is a dense web of connections that are unequal, dynamic, and deeply interdependent, traditional economic models often struggle to capture some parts of this complexity. However, mathematical tools, especially those of graph theory, offer a different way to see and measure what's really happening beneath the surface. Graph theory allows us to treat countries as nodes and trade flows as directed links, making it possible to model the entire system through a mathematical perspective. This paper is going to explore how graph theory can serve as such a tool to help quantify and explain global trade and changes.

In addition, this paper also explores how dynamic changes in trade taxes can impact the structure of the global trade network using a mathematical model because international trade is more than just the exchange of common goods. Variables like trade agreements, sanctions, pandemics, and the current hot topic, tariffs, can all significantly alter the flow of goods and the economic trading landscape. These factors heavily influences this intricate web of trade.

With events such as the recent proposition of U.S. Tariffs, which our model will focus on, we can observe how trade centrality shifts over time by running our model recursively for many cycles. Thus, this model not only captures immediate effects, such as reduced imports or exports, but also long-term structural changes in global economic network.

Understanding the consequences of these changes and variables is immensely important for researchers and policymakers. For instance, tariffs are sometimes employed to adjust trade imbalances or protect domestic industries. However, a single policy decision can send ripples through the entire network. Some aspects of these moves and effects aren't always easy to see especially with traditional economic models. By analyzing trade networks through the lens of mathematics, we can better understand how policy decisions reshape the structure of the global economy, not just in theory, but also in measurable and sometimes unexpected ways.

2 Preliminaries

To apply graph theory to the modeling of international trade, we must first establish the fundamental definitions of theoretical graph structures. This section introduces the formal concepts of graphs, nodes, edges, other types of graphs, and other concepts relevant to our analysis.

2.1 Graphs

A **graph** can be used as a complex framework for many difficult tasks in many fields of study, but fundamentally, a graph is simply a system of connections of objects represented by a collection of **nodes** or **vertexes** with **edges** connecting them. Figure 1 shows a simple graph with a couple of nodes and edges.

Definition 1. *Formally we define a finite* ***graph*** *as an ordered triple*

$$G = (V, E, w),$$

where:

- $V = \{v_1, v_2, \ldots, v_n\}$ *is a finite set of* vertices *or* nodes*,*
- $E \subseteq V \times V$ *is a set of ordered pairs representing* edges*,*

- *$w : E \to \mathbb{R}_{\geq 0}$ is a weight function assigning a non-negative real value to each edge. [1]*

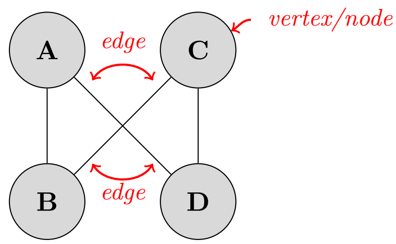

Figure 1: Graph depicting nodes/vertexes and edges

Graphs can be categorized based on the structure of their set of edges. A **graph is undirected** if edges are defined as unordered pairs:

$$E \subseteq \{\{v_i, v_j\} : v_i, v_j \in V,\ v_i \neq v_j\}.$$

Here, the edge $\{v_i, v_j\}$ represents a two way connection between v_i and v_j, and satisfies $\{v_i, v_j\} = \{v_j, v_i\}$ by definition. Thus symmetry is encoded directly into the edge structure, and no directionality is implied. In simpler words, in an undirected graph, edges don't have directions, an edge merely shows that two nodes are connected. In the case of the global trade network, it would only show that two countries are connected but regardless of the importer or exporter. Figure 1 is an example of a simple undirected graph, with labeled nodes and edges.

In contrast, in a **directed graph**, the edges are defined as a subset of ordered pairs:

$$E \subseteq V \times V,$$

so that each edge $(v_i, v_j) \in E$ represents a one way connection from node v_i to node v_j with direction. The order of the pair matters in this case because (v_i, v_j) and (v_j, v_i) are considered distinct edges.

Additionally, graphs may also be **weighted**, meaning that each edge has an assigned numerical value. Formally, a graph is weighted if there is a function

$$w : E \to \mathbb{R}_{\geq 0}$$

This function assigns a non negative real number $w(e)$ to every edge $e \in E$. This weight typically represents the strength, cost, capacity, or frequency of the connection, depending on the context of the graph.

In our model, the graph is both directed and weighted. Each directed edge $(v_i, v_j) \in E$ is assigned a weight $w(v_i, v_j)$ corresponding to the volume of goods exported from node v_i (the exporter) to node v_j (the importer) based on value. A larger weight indicates a greater trade flow between the two countries. If no such trade occurs, the corresponding edge is omitted or assigned a weight of zero.

Figure 2 shows an example of a directed, weighted graph.

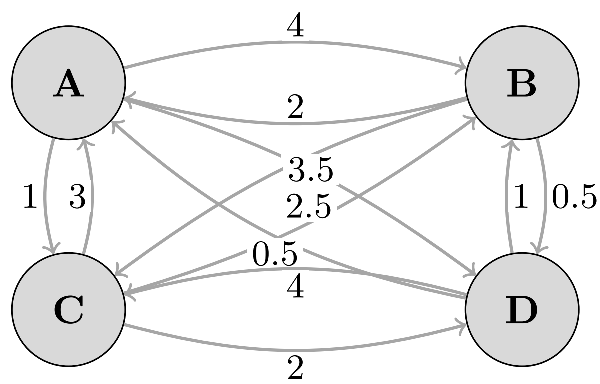

Figure 2: Directed, weighted graph with illustrative edge weights

2.2 Matrices and Graph Representation

A **matrix** is simply just a rectangular array of numbers arranged in rows and columns.

Definition 2 ([1])**.** *Formally, an $m \times n$ matrix is a collection of real numbers written as follows:*

$$M = \begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{m1} & m_{m2} & \dots & m_{mn} \end{bmatrix} \in \mathbb{R}^{m \times n}, \tag{1}$$

where m_{ij} represents the entry in the i-th row and j-th column. Matrices are fundamental tools in mathematics and are used to represent data, systems of equations, and more generally, linear transformations.

In the context of graph theory, matrices provide a powerful way to represent the structure of a graph through numbers. We will numerical values to the connections between nodes and convert a graph into a matrix that captures the relationships between its vertices in a precise and structured form to analyze mathematically.

Definition 3. *For a directed, weighted graph $G = (V, E, w)$ with n nodes, we define the* ***adjacency matrix*** *$A \in \mathbb{R}^{n \times n}$, where each entry A_{ij} records the weight of the edge from node v_i to node v_j. That is:*

$$A_{ij} = \begin{cases} w(v_i \to v_j) & \textit{if } (v_i, v_j) \in E, \\ 0 & \textit{otherwise.} \end{cases}$$

[1]

Each row i of the matrix corresponds to a exporter node, and each column j corresponds to a importer node. Table 1 offers a visual representation of the relation that the matrix represents.

If no edge exists from node v_i to v_j, then $A_{ij} = 0$. Furthermore, in our trade network model, we assume that $A_{ii} = 0$ for all i because countries do not trade internationally with themselves. The adjacency matrix will then serve as a numeric representation of the entire graph structure and is the starting point for defining trade flow probabilities. Table1 provides an a visual guide to how our adjacency matrix would look.

	Australia	**Brazil**	**Canada**
Australia	Export from Australia to Australia (0)	Export from Australia to Brazil	Export from Australia to Canada
Brazil	Export from Brazil to Australia	Export from Brazil to Brazil (0)	Export from Brazil to Canada
Canada	Export from Canada to Australia	Export from Canada to Brazil	Export from Canada to Canada (0)

Table 1: Export flows between Australia, Brazil, and Canada

2.3 Transition Matrices

While the adjacency matrix A structures the raw edge weights of a graph, it does not yet represent a probability structure. To study the dynamics of flow through the graph, like a random walk, we must construct a **transition matrix**, which captures the chance of moving from one node to another through a matrix.

Definition 4 ([1, 2])**.** *Given an adjacency matrix $A \in \mathbb{R}^{n\times n}$, we define the **transition matrix** $P \in \mathbb{R}^{n\times n}$ by normalizing each row of A so that it sums to 1. Formally, the entries of P are given by:*

$$P_{ij} = \frac{A_{ij}}{\sum_{k=1}^{n} A_{ik}}.$$

In the structure of a transition matrix, each row i of P represents a probability distribution over the possible destinations from source node v_i. In our trade context, for a given exporter i, P_{ij} gives the probability that a good will be traded to importer j. Thus, each row of P satisfies:

$$\sum_{j=1}^{n} P_{ij} = 1.$$

The transition matrix P transforms the graph into a Markov-like process, where each node represents a state, and each entry P_{ij} shows the transition probability from node i to node j [2]. This way of representation enables the use of probabilistic tools to analyze long-term behavior on the graph. Matrix 2 shows a simple example of a transition matrix, with probabilities summing up to one.

$$P = \begin{bmatrix} 0.70 & 0.10 & 0.20 \\ 0.20 & 0.50 & 0.30 \\ 0.30 & 0.30 & 0.40 \end{bmatrix} \tag{2}$$

In the following sections, we use this transition matrix to define a random walk on the graph, and ultimately to compute the *stationary distribution*, a long-term stable state of values that captures each country's trade centrality in the global network.

2.4 Random Walks

A **random walk** is a fundamental stochastic, or random, process defined on a graph, in which a "walker" moves from node to node based on the transition probabilities on the transition matrix. In our setting, this movement models the flow of trade through the network of countries.

Definition 5. *To formally to define a random walk, let $G = (V, E, w)$ be a directed, weighted graph with $n = |V|$, and let $P \in \mathbb{R}^{n\times n}$ be the transition matrix normalized by row as defined earlier. A random walk on G is basically simulating a discrete-time Markov chain $\{X_t\}_{t\in\mathbb{N}}$ where:*

$$\mathbb{P}(X_{t+1} = v_j \mid X_t = v_i) = P_{ij}.$$

[2]

This means that, at each time step t, the walk moves from node v_i to node v_j with probability P_{ij}.

Intuitively in our context of international trade, a random walk on this graph simulates the probabilistic movement of a unit of trade through the network. The probability of transitioning from country v_i to country v_j is determined by the ratio of exports from v_i allocated to v_j compared the rest of the nations, as given by the transition matrix P.

Countries with a larger number of export destinations will have more connecting to other nodes, and those that receive a greater share of trade volume from a given exporter will correspond to higher transition probabilities. For example, if the random walk of a trade unit is currently at the node representing China, the probability of transitioning to the node representing the United States is relatively high, since the United States is one of China's largest export partners.

The initial distribution of the walker is given by a row probability vector $\mu^{(0)} \in \mathbb{R}^{1\times n}$, with P as the transition matrix. The distribution after t steps is given by

$$\mu^{(t)} = \mu^{(0)} P^t.$$

[2] Random walks capture how visitation probability moves through the network over time. If the Markov chain induced by P is irreducible and aperiodic, then $\mu^{(t)}$ converges to a unique limiting distribution independent of the initial distribution. This leads us to the concept of the stationary distribution, which we define next.

2.5 Stationary Distribution

Definition 6. *Let $P \in \mathbb{R}^{n\times n}$ be a row-stochastic transition matrix on the finite state space $V = \{v_1, v_2, \ldots, v_n\}$. We define a **stationary distribution** as a column probability vector $\pi \in \mathbb{R}^n$ satisfying*

$$P^{\mathsf{T}}\pi = \pi, \qquad \sum_{i=1}^{n} \pi_i = 1, \qquad \pi_i \geq 0 \quad \text{for all } i.$$

Equivalently, the corresponding row vector π^{T} satisfies

$$\pi^{\mathsf{T}} P = \pi^{\mathsf{T}}.$$

This convention is consistent with the row-vector evolution equation

$$\mu^{(t)} = \mu^{(0)} P^t.$$

If the walk begins with distribution π^{T}, one further transition leaves the distribution unchanged:

$$\pi^{\mathsf{T}} P = \pi^{\mathsf{T}}.$$

In this trade-network model, π_i is the long-run visitation probability of country i under transitions defined by bilateral destination shares. A larger π_i indicates greater stationary centrality within the selected network. Because each row of A is normalized, multiplying every entry in one exporter's row by the same positive constant leaves P unchanged. Consequently, π_i does not directly measure a country's total trade volume, GDP, or economic size; it measures centrality induced by the relative destination-share structure and indirect network connectivity.

When the transition matrix changes under a modeled policy scenario, comparing the resulting stationary distributions provides a way to examine the sensitivity of this network-centrality measure.

2.6 Computing the Stationary Distribution

For a square matrix $M \in \mathbb{R}^{n\times n}$, a nonzero vector $x \in \mathbb{R}^n$ is a right eigenvector with eigenvalue λ when

$$Mx = \lambda x.$$

[1]

For the row-stochastic matrix P, the stationary column vector satisfies

$$P^{\mathsf{T}}\pi = \pi.$$

Thus, π is a right eigenvector of P^{T} associated with eigenvalue 1, or equivalently a left eigenvector of P. Rewriting the stationary equation gives

$$\left(P^{\mathsf{T}} - I\right)\pi = 0.$$

Because $P\mathbf{1} = \mathbf{1}$, the value 1 is an eigenvalue of P, and therefore also of P^{T}. The relevant characteristic equation may be written as

$$\det\left(P^{\mathsf{T}} - \lambda I\right) = 0.$$

The existence of eigenvalue 1 alone does not guarantee a unique stationary distribution or convergence from every initial distribution. For a finite Markov chain, irreducibility guarantees a unique stationary distribution with strictly positive entries. If the chain is also aperiodic, then

$$\mu^{(0)}P^t \longrightarrow \pi^{\mathsf{T}}$$

for every initial probability row vector $\mu^{(0)}$ [2, 4].

The stationary vector can be found by solving

$$\left(P^{\mathsf{T}} - I\right)\pi = 0$$

together with

$$\sum_{i=1}^{n} \pi_i = 1.$$

For the 20-country network, we compute this vector numerically in MATLAB.

3 Methodology

My study uses the understanding of all of these concepts to model the trade network of 20 economically significant countries with available data on the internet. All of the data processing, matrix manipulation, and simulations will be carried out with MATLAB.

3.1 Data Collecting

Trade data is collected from public data sources such as the UN Comtrade database [3]. The dataset contains the export from from each country to the rest of the selected countries. The dataset was then processed and formatted into excel sheets.

With the assistance of Matlab, the raw data was then processed from the excel sheets to form a country-to-country export import matrix by using *readmatrix()*. zeros were added in the place of missing export values from one country to itself. The matrix though only included the export value, intuitively represents the import of each country from the rest.

3.2 Transition Matrix and Stationary Distribution Calculation

The matrix A represents the weighted edge set of the directed trade graph, where A_{ij} is the value of exports from country i to country j. We construct the row-stochastic transition matrix P by normalizing each row of A:

$$P_{ij} = \frac{A_{ij}}{\sum_{k=1}^{n} A_{ik}},$$

provided that every row has a positive sum. Each row i then represents the distribution of exporter i's trade among the selected destination countries, and

$$\sum_{j=1}^{n} P_{ij} = 1.$$

The long-run visitation behavior of this row-normalized network is described by the stationary column vector π, which satisfies

$$P^{\mathsf{T}}\pi = \pi.$$

We compute it numerically in MATLAB as follows:

```
rowSums = sum(A, 2);
assert(all(rowSums > 0), ...
    'Every exporter must have a positive row sum.');

P = A ./ rowSums;
assert(max(abs(sum(P, 2) - 1)) < 1e-12, ...
    'P must be row-stochastic.');

[V, D] = eig(P.');
[~, index] = min(abs(diag(D) - 1));

pi = real(V(:, index));
if sum(pi) < 0
    pi = -pi;
end
pi = pi / sum(pi);

assert(norm(P.' * pi - pi, 1) < 1e-10, ...
    'Computed vector is not stationary.');
assert(abs(sum(pi) - 1) < 1e-10, ...
    'Stationary probabilities must sum to one.');
assert(all(pi >= -1e-10), ...
    'Stationary probabilities must be nonnegative.');
```

Here, D contains the eigenvalues of P^{T}, and the columns of V are the corresponding right eigenvectors of P^{T}, equivalently the left eigenvectors of P. We select the eigenvector associated with the eigenvalue numerically closest to 1, choose its sign so that its sum is positive, and normalize it to sum to 1. The residual check verifies the stationary equation $P^{\mathsf{T}}\pi = \pi$.

The same calculation is applied after each tariff adjustment. Comparing the resulting vectors measures changes in stationary visitation centrality within the selected, row-normalized network.

3.3 Dynamic US Tariff Simulation with Recursive Adjustment

To examine how a simplified tariff adjustment changes the network, let $A^{(k)}$ denote the bilateral trade matrix at cycle k. For each country j, define

$$E_j^{(k)} = A_{j,\mathrm{US}}^{(k)}$$

as exports from country j to the United States, equivalently U.S. imports from j, and define

$$I_j^{(k)} = A_{\mathrm{US},j}^{(k)}$$

as imports by country j from the United States, equivalently U.S. exports to j.

For $I_j^{(k)} > 0$, the simulation uses the simplified rule

$$\tau_j^{(k)} = \max\left(0.10, \frac{1}{2}\frac{E_j^{(k)} - I_j^{(k)}}{I_j^{(k)}}\right).$$

This rule is a modeling assumption inspired by U.S. tariff policy; it is not an estimated causal relationship between tariff rates and trade volumes.

The U.S.-bound trade entry is adjusted according to

$$A_{j,\mathrm{US}}^{(k+1)} = A_{j,\mathrm{US}}^{(k)}\left[1 - \left(\tau_j^{(k)} - \tau_j^{(k-1)}\right)\right],$$

while the other entries follow the assumptions of the simulated scenario. Increasing τ_j therefore reduces the modeled edge from country j to the United States, not the edge from the United States to country j.

After updating $A^{(k+1)}$, we row-normalize it to obtain $P^{(k+1)}$ and compute the stationary column vector from

$$\left(P^{(k+1)}\right)^{\mathsf{T}} \pi^{(k+1)} = \pi^{(k+1)}.$$

The updated trade matrix $A^{(k+1)}$, rather than the stationary distribution, determines the bilateral quantities $E_j^{(k+1)}$ and $I_j^{(k+1)}$ used in the next cycle. The stationary distribution is an output that summarizes the destination-share network after each adjustment; it is not itself converted into dollar-valued trade balances.

Because row normalization rescales each exporter's adjusted links into relative destination shares, the simulation should be interpreted as a network-sensitivity exercise under the stated update rule. It does not independently estimate behavioral elasticities or forecast actual future trade volumes.

4 Analysis of Data

4.1 Initial Distribution

Table 2: Initial stationary distribution $\pi^{(0)}$ for all 20 countries.

Country	AU	BR	CA	CN	EG	FR	DE
$\pi^{(0)}$	0.0244	0.0203	0.0741	0.1389	0.0502	0.0543	0.0208
Country	IN	ID	IT	JP	MX	NG	SA
$\pi^{(0)}$	0.0383	0.0649	0.0414	0.0695	0.0046	0.0158	0.0474
Country	ZA	KR	TH	TR	UK	US	
$\pi^{(0)}$	0.0143	0.0208	0.0169	0.0158	0.0356	0.2316	

The table above (Table 2) shows the initial stationary distribution $\pi^{(0)}$ for all 20 countries in the network, arranged horizontally for comparison.

To begin our analysis, we first examine the initial stationary distribution $\pi^{(0)}$, computed from the unadjusted row-stochastic matrix P. Table 2 reports each country's stationary visitation probability within the selected 20-country network. The United States has the largest modeled value, approximately 0.232, followed by China at 0.139, Canada at 0.074, Japan at 0.069, and Indonesia at 0.065.

These values describe network centrality under relative destination-share transitions; they are not shares of total world trade and do not directly rank countries by absolute export volume, GDP, or economic size. Lower or higher values arise from the pattern of incoming destination shares and indirect connectivity among the 20 included countries. Omitting important partners can materially change those shares. For example, Germany's value may be affected by the exclusion of several major European trading partners from the selected network.

Overall, this initial distribution sets the benchmark for understanding how tariffs and policy changes affect the relative influence of countries in the global network. By comparing future stationary distributions after tariff cycles to $\pi^{(0)}$, we can quantify how trade centrality shifts between major and minor players.

Now that we have established the trade centrality through the initial stationary distribution $\pi^{(0)}$, we can now investigate how this distribution evolves under tariff-caused changes. By recursively applying the U.S. tariff adjustment rule for every country to the trade matrix and recomputing the stationary distribution at each cycle, we can see how the relative influence of countries shifts over cycles.

4.2 The US's Centrality after Tariffs on All Countries

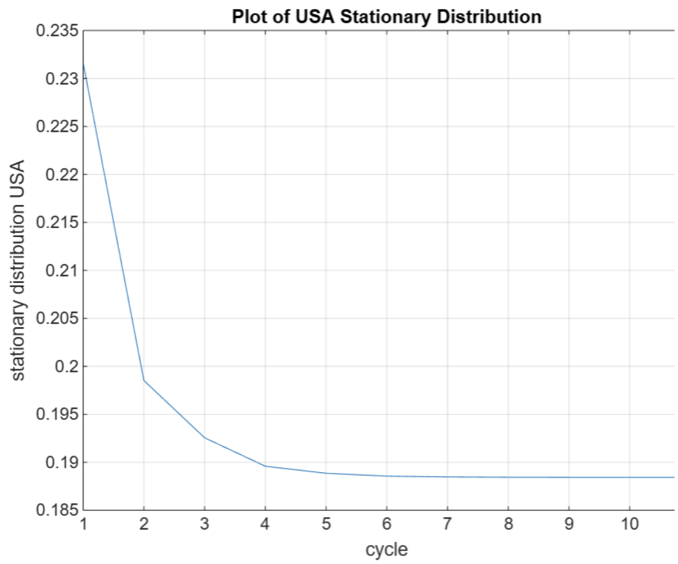

Figure 3: Data Plot of the US's Stationary Distribution Over Ten Cycles

The plot in figure 3 above shows the evolution of the stationary distribution value for the United States across 10 recursive tariff adjustment cycles. Initially, in cycle 1, the U.S. holds a relatively high trade centrality of approximately 0.232, the highest, which indicates its strong position within the global trade network.

However, after the first round of tariff application, where the U.S. imposes increased import taxes on countries with high trade surpluses, its own centrality drops significantly. By cycle 2, the stationary distribution falls below 0.20, reflecting the immediate negative feedback effect of tariff-induced trade contraction.

Subsequent iterations show a lessened decline. From cycles 3 through 6, the U.S. centrality gradually converges toward a lower equilibrium, settling around 0.188. This suggests that the trade network responds non-linearly to tariff changes. While initial shifts are sharp, further rounds produce diminishing marginal impacts as the system approaches a steady state.

Under the simulation's stated adjustment and normalization rules, the U.S. stationary centrality declines rather than increases. The flattening of the curve indicates that this particular numerical iteration approaches a fixed point; it should not be interpreted as evidence that the real global economy necessarily reaches the same equilibrium.

4.3 China's Centrality after Tariffs on All Countries

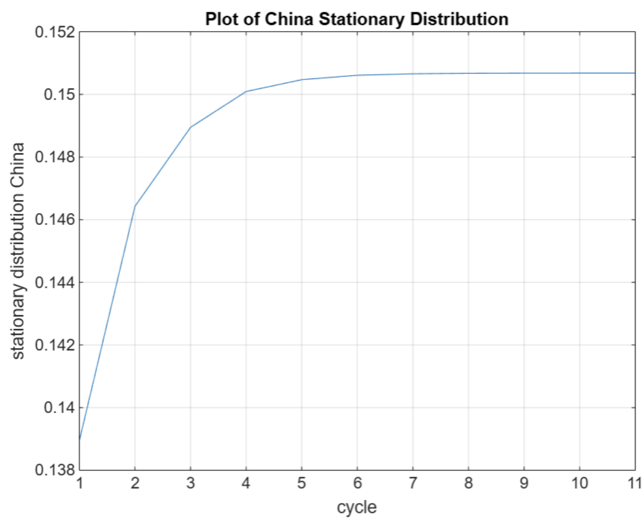

Figure 4: Data Plot of the China's Stationary Distribution Over Ten Cycles

In contrast to the declining trend observed for the United States, the stationary distribution of China, who holds the second highest distribution score, increases steadily across the simulation cycles, as shown in figure 4. Beginning at approximately 0.139 in cycle 1, China's trade centrality rises to over 0.151 by cycle 6 and eventually stabilizes. This upward trajectory indicates that, despite being the primary target of U.S. tariff policies, China becomes increasingly central in the global trade network as the simulation progresses.

Within the model, reducing U.S.-bound links and then row-normalizing the adjusted matrix raises the relative destination shares of some non-U.S. partners. This reweighting can increase China's stationary centrality even when China is directly targeted. The simulation does not independently establish that real nations redirect additional dollar-valued trade toward China.

Under the model's assumptions, a policy intended to reduce China's role can instead increase China's relative stationary centrality. This counterintuitive result is a property of the specified network update and motivates further empirical testing; it is not by itself a prediction that the same outcome will occur in observed trade.

4.4 Global Changes in Trade Centrality

While the U.S. and China shows the most significant shifts under the recursive tariff adjustments, it is very important to examine the broader effects on all 20 countries in the network. Figure 5 presents the stationary probabilities for all countries across the 10 simulation cycles. Overall, most countries experience

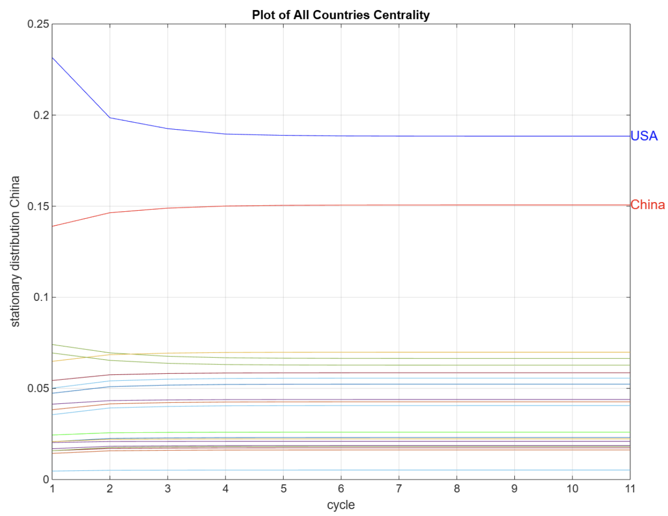

Figure 5: Stationary distributions of all 20 countries over 10 recursive tariff adjustment cycles.

only minor changes in their stationary probabilities relative to the larger movements for the U.S. and China. Most show slight increases in modeled centrality as row normalization changes the relative destination shares after U.S.-bound links are reduced, while a few show small declines through indirect network effects.

Despite these minor adjustments, the global ranking of countries remains relatively stable throughout the cycles, with the U.S. and China retaining their positions as the top two nodes in the network. This suggests that while tariffs can significantly influence the relative trade dominance of the largest economies, the structural relationships among mid- and lower-ranked economies remain comparatively robust.

4.5 Scenario: U.S. Tariffs Targeting Only China

Assuming that China is widely considered the primary target of the ongoing U.S.–China trade tensions, we simulate a scenario where the U.S. imposes tariffs exclusively on imports from China, leaving its trade relationships with all other countries unchanged. This allows us to isolate the bilateral effects of the trade war and analyze how a tariff strategy focused on china will influences the global trade network. The resulting stationary distributions over 10 recursive cycles are shown in Figure 6.

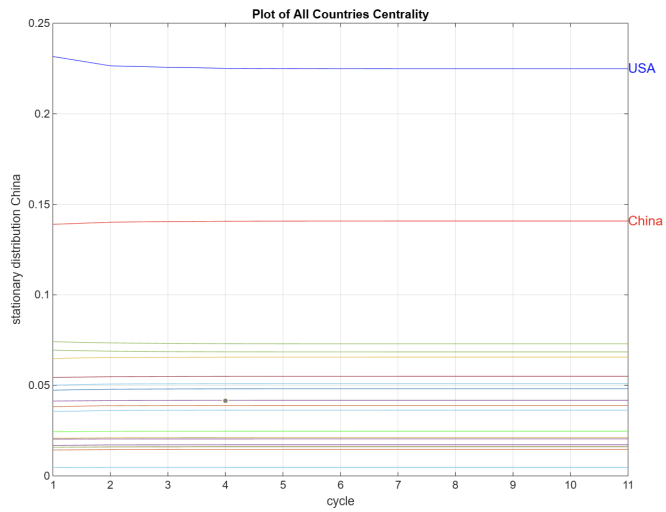

Figure 6: Stationary distributions of all 20 countries over 10 recursive tariff adjustment cycles when only China is targeted by U.S. tariffs.

In this scenario, the overall pattern remains similar to the global tariff case, but the magnitude of changes is notably smaller. The U.S. still experiences a decline in its centrality, though the drop is less severe as in the broader tariff scenario. China's centrality still rises slightly despite being the direct target of the modeled tariff. In this simulation, the relative reweighting of non-U.S. destination shares partially offsets the reduced China-to-U.S. link; the model does not directly estimate additional trade volumes through intermediary partners.

For the other 18 countries in the network, the shifts in stationary distribution are very small. Some countries, particularly those with strong ties to both the U.S. and China, experience slight differences as trade flows bypass direct tariffs. However, these changes are minor compared to the substantial redistribution observed when the U.S. imposes tariffs across the entire network.

5 Conclusion

This study presents a simplified framework for representing selected bilateral trade flows as a directed network and measuring stationary visitation centrality through a row-stochastic transition matrix. Under this convention, the stationary column vector is correctly defined by

$$P^{\mathsf{T}}\pi = \pi.$$

The entries of π summarize long-run visitation probabilities generated by relative destination shares and indirect connectivity within the selected network. They do not directly measure absolute trade volume, GDP, or economic size.

The tariff simulation adjusts U.S.-bound entries of the trade matrix, row-normalizes the updated matrix, and recomputes the stationary distribution over repeated cycles. The bilateral export and import quantities used by the tariff rule are derived from the updated trade matrix A; the stationary vector π is an output centrality measure rather than a mechanism for updating dollar-valued trade balances.

Under the assumptions of the simulation:

- the U.S. stationary centrality decreases after the modeled tariff adjustments;
- China's stationary centrality increases as the row-normalized destination-share structure changes, even when China is directly targeted;
- most other countries experience smaller changes, and the broad centrality ordering remains comparatively stable; and
- local changes to U.S.-bound links can propagate through the stationary distribution of the full selected network.

These outcomes demonstrate how a network model can generate counter-intuitive comparative-statics results. However, the model does not estimate behavioral trade elasticities, independently model the creation of new bilateral flows, or constitute an empirical forecast. Its appropriate use is as a sensitivity analysis that illustrates how the specified tariff-update and normalization assumptions affect stationary network centrality. Future work could add empirically estimated responses, broader country coverage, robustness checks, and out-of-sample validation.

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